M.Sc. Mathematics First Semester

(Faculty of Science)
MATHEMATICS
FIRSTPAPER
Algebra-I
Assignment

Unit - 1

1. State and prove Fundamental theorem of Homomorphism.

2. Let G_1 and G_2 be two groups. If N_1 and N_2 are normal subgroups to G_1 and G_2 $resp.$,then

   $N_1 * N_2$ is normal subgroups to $G_1 * G_2$ and

$$\frac{G_1 * G_2}{N_1 * N_2} \cong \frac{G_1}{N_1} * \frac{G_2}{N_2}$$

Unit - 2

3. Let H be a subgroup of G, then $H^i \subset G^i$, for all i ∈ N.

4. If G is solvable group , then every subgroup of G is also solvable.

Unit - 3

5. Let K/F be a field extension. Then an element a ∈ K is algebraic over F if and only if F(a)

   is finite extension of F, i.e. $[\, F(a) \colon F]$ is finite.

6. If F is a field,then every polynomial f(x) ∈ F[x] has a splitting field.

Unit - 4

7. If K is a finite extension of a field F, then the group G(K/F) of F-automorphisms of K is finite and

$$O[G\left(\frac{K}{F}\right)] \leq [K \colon F]$$

8. State and prove the fundamental theorem of Galois theory.

M.Sc. Mathematics First Semester
Second Paper
Real Analysis

Unit -I

Q.1 Prove that outer measure of an interval is equal to its length.

Q.2 Let X be a non - empty subset of R and let α be any collection of subsets of X . Then prove that there is a smallest algebra A^* containing α .

Unit -II

Q.3 Let $\{f_n\}$ be a sequence of measurable functions define on a measurable set E, which converging measure to the function f , then prove that their exist is subsequence $\{f_{n_k}\}$which convergence to f almost everywhere on E.

Q.4 Define equivalent functions. Let f and g be two functions defined on a measurable set E . If f is a measurable function on E and g is equivalent to f , then prove that g is also measurable function on E.

Unit -III

Q.5 Let f is be a bounded and measurable function define on a measurable set E, then prove that f is lebesgue integrable on E .

Q.6 Let f be a bounded and measurable function defined on a measurable set E and let $f(x) \geq 0$ almost everywhere on E. If $\int_E f(x) = 0$, then prove that $f(x) = 0$ almost everywhere on E.

Unit -IV

Q.7 Let f be a summable function define on a measurable set E. Then prove that for a given $\epsilon > 0$, there exist a $\delta > 0$ such that

$$\left|\int_e f(x)dx\right| < \epsilon$$

where e is a measurable subset of E with $m(e) < \delta$.

Q.8 A series $\sum_{i=1}^{\infty} f_i$ of pairwise orthogonal elements in L_2 is convergent iff the series of real numbers $\sum_{i=1}^{\infty} ||f_i||^2$ is convergent.

M.Sc. Mathematics First Semester
Paper – III (DIFFERENTIAL EQUATION- I)
Assignment

Unit -I

1.a) Solve the following differential equation

$$cosy\left(\frac{d^2y}{dx^2}\right)-siny\left(\frac{dy}{dx}\right)^2+cosy\left(\frac{dy}{dx}\right)=x+1$$

b) Convert the following differential equation into Riccati's equation and then solve.

$$2x^2\left(\frac{dy}{dx}\right)=(x-1)(y^2-x^2)+2xy$$

2 a) Solve:

$$y(1-log(y))\left(\frac{d^2y}{dx^2}\right)+(1+log(y))\left(\frac{dy}{dx}\right)^2=0$$

b) Solve:

$$\frac{dy}{dx}=2+\frac{1}{2}\left(\frac{x-1}{x}\right)y-\frac{1}{2}y^2$$

given that $\left(x+\frac{1}{x}\right)$ is a particular solution of it.

(c) Solve the differential equation

$$x^2y\frac{d^2y}{dx^2}+\left(x\frac{dy}{dx}-y\right)^2-3y^2=0$$

Unit-II

3. Solve

a) $(x^2y-y^3-y^2z)dx+(xy^2-x^2z-x^3)dy+(xy^2+x^2y)dz=0$

b) $z(y+z)dx+z(t-x)dy+y(x-t)dz+y(y+z)dt=0.$

Q.4 Solve

(a) Solve by method of Homogeneous Equation:

$$\left(y\,z+z^2\right)dx-\left(xz\right)dy+\left(xy\right)dz=0$$

(b) Solve by method of Auxiliary Equation:

$$z\left(z-y\right)dx+\left(z+x\right)zdy+x\left(x+y\right)dz=0$$

Unit-III

5. Solve $2x(1-x)\frac{d^2y}{dx^2}+(1-x)\frac{dy}{dx}+3y=0$ in power series.

6. a) solve the hypergeometric equation

$$\frac{d}{dx}\left[(1-x^2)\frac{dy}{dx}\right]+n(n+1)y=0$$ in series.

b) Solve in series

$$x^4\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x^{-1}$$

Unit-IV

7. a) Solve:

$$t-sec^4y\times r=2tany\times q$$

b) Solve:

$$s^2-rt=a^2$$

8. Solve by Monge's method:

a) $z(1+q^2)r-\ 2pqzs(1+p^2)t\ +\ z^2\ (rt-s^2)+1+p^2\ +\ q^2=0$

b) $qr+(p+x)s+yt+y(rt-s^2)+q=0$

M.Sc. (Mathematics) I Semester

Paper-IV (Differential Geometry)

Assignment

UNIT-I

Q1. State and prove Serret-Frenet formulae.

Q2. For a point of the curve of intersection of the surfaces $x^2 - y^2 = c^2$, $y = x\tanh\frac{z}{c}$. Show that $\rho = -\sigma = \frac{2x^2}{c}$.

UNIT-II

Q3. Find the equation to the inflexional tangents through a point (x, y, z) of the surface $\eta 2\zeta = 4c\xi$.

Q4. Derive necessary and sufficient condition that a surface $\rho = f(\xi, \eta)$ must represent a developable surface.

UNIT-III

Q5. Find the Gaussian curvature at the point (u, v) of the anchor ring $x = (b + a\cos u)\cos v$, $y = (b + a\cos u)\sin v$, $z = a\sin u$, where the domain of u, v is $0 \le u \le 2\pi$, $0 \le v \le 2\pi$. Verify that the total curvature of the whole surface is zero.

Q6. Prove that for the helicoid $x = u\cos\theta$, $y = u\sin\theta$, $z = c\theta$, $\rho_1 = -\rho_2 = \frac{u^2 + c^2}{c}$, where $u^2 = x^2 + y^2$, and that the lines of curvature are given by $d\theta = \pm\frac{du}{\sqrt{u^2 + c^2}}$.

UNIT-IV

Q7. Prove that for the surface $x = 3u(1 + v^2) - u^3$, $y = 3v(1 + u^2) - v^3$, $z = 3u^2 - 3v^2$, the asymptotic line are $u + v =$ constant.

Q8. For the conoid $z = f\left(\frac{y}{x}\right)$, prove that the asymptotic lines consist of the generators and the curves whose projections on the xy-plane are given by $x^2 = af'\left(\frac{y}{x}\right)$, where a is an arbitrary constant.

M.Sc. First Semester
MATHEMATICS
FIFTH PAPER
Dynamics of Rigid Bodies

1. (a) State and prove D' Alembert's Principle.
   (b) A rough uniform board, of mass 'm' in and length 2a, rests on a smooth horizontal plane and a man of mass M walks on it from one end to the other. Find the distance through which the board moves in this time.

2. (a) A uniform rod, of mass m and length $2a$, can turn freely about one end which is fixed, it is started with angular velocity w from the position in which it hangs vertically ; find its angular velocity at any instant and time of describing an angle θ .
(b) A rod oA which is uniform, of length 2a, free to turn about its end 0, revolves with uniform angular velocity ω about the vertical oZ through o, and is inclined at a constant angle $\propto$ $to\ oZ$, show that the value of $\propto$ is either zero or $cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$.

3. (a) A solid homogeneous cone, of height h and vertical angle 2 $\propto$ oscillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{h}{5}(4+tan^2 \propto)$.
   (b) Find the position of the centre of percussion in the following cases:
   (i) A uniform circular plate, axis a horizontal tangent.
   (ii) A uniform rod with one-end fixed.

4. (a) Two equal similar rods AB and BC are freely hinged at B and lie in a straight line on a smooth table. The end A is struck by a blow perpendicular to AB. Show that the resulting velocity of A is $3\frac{1}{2}$ times that of B.
   (b) A cylinder rolls down a smooth plane whose inclination to the horizontal is $\propto$, unwrapping, as it goes, a fine string fixed to the highest point of the plane. Find its acceleration and the tensions of the string.

5.(a) Deduce Euler's Geometrical equations for the motion of a rigid body with respect to a fixed point on it.
   (b) A body under the action of no forces moves so that the resolved part of its angular velocity about one of the principal axes at the centre of gravity is constant. Show that the angular velocity of the body must be constant.

6. (a) A lamina in the form of an ellipse is rotating in its own plane about one of its foci with angular velocity ω. This focus is set free and the other focus, at the same instant is fixed. Show that the ellipse now rotates about it with angular velocity $\omega\left(\frac{2-5e^2}{2+3e^2}\right)$.
   (b) A small insect moves along a uniform bar, of mass equal to itself and of length $2a$, the ends of which are constrained to remain on the circumference of a fixed circle whose radius is $\frac{2a}{\sqrt{3}}$. If the insect starts from the middle point of the bar and moves along the bar with

relative velocity V; show that the bar in time t will turn through an angle $\frac{1}{\sqrt{3}}\left(tan^{-1}\frac{Vt}{a}\right)$.

7. (a) Explain how Lagrange's Equation are used in case of small oscillations.

(b) A circular disc, of radius, has a thin rod pushed through its centre perpendicular to its plane, the length of the rod being equal to the radius of the disc. Show that the system cannot spin with the rod vertical unless the angular velocity is greater than $\sqrt{\left(\frac{20g}{a}\right)}$.

8. (a) Write Hamilton's equations for a particle of mass m moving in a plane under a force which is some function of the distance from the origin.

(b) A particle moves on a smooth curve, joining the two fixed points A and B, under gravity starting from rest from A. Find the form of the path in order that the time from A to B is minimum.

M.Sc. Third Semester Assignment
MATHEMATICS
SIXTH PAPER
Calculus of Variation & Special Function-I

1.(a) Find the extremals for the integrals defined by:
$I = \int_0^1 (y^2 + (y')^2 + 2ye^x)dx$passing through (0,0) and (1, e)

(b) Derive four special cases of Euler – Lagrange equation as:
(i) f is independent of y
(ii) f is independent of x
(iii) f is independent of y'
(iv) f is independent of x and y

2.(a) Find the shortest curve joining two points (x_1, y_1) and (x_2, y_2) by the use of variational method.

(b) Find the extremals of the functional $\int_a^b \frac{(y')^2}{x^3} dx$.

3.(a) Find the extremals of the functional

$$I[y(x), z(x)] = \int_0^{\pi/2} [(y')^2 + (z')^2 + 2yz]dx$$

withthe boundary conditions

$$y(0) = 0, \; y(\pi/2) = -1; \quad z(0) = 0, \; z(\pi/2) = 1$$

(b) Discuss the variational problem involving several higher order derivatives.

4.(a) Show that the curve of shortest distance (geodesic) on a right circular cylinder is a helix or a generator.

(b) Find the extremal of the functional $I = \int_0^1 (1 + (y'')^2)dx$ with conditions

$$y(0) = 0, \; y'(0) = 1, \; y(1) = 1, \; y'(1) = 1$$

5.(a) Find the series solution of Gauss's hyper geometric equation in the neighbourhood of $x = 0$.

(b) Prove that

$$ {}_2F_1 \; (\alpha, \beta; 2\beta; z) = \frac{\left(1 - \frac{1}{2}z\right)^{-\alpha}}{2^{2\beta-1} \cdot B(\beta, \beta)} \int_0^{\pi/2} (\sin\phi)^{2\beta-1} \begin{bmatrix} \{1 + \mathcal{G}\cos\phi\}^{-\alpha} \\ +\{1 - \mathcal{G}\cos\phi\}^{-\alpha} \end{bmatrix} d\phi$$

where$\mathcal{G} = \frac{z}{2-z}$

6.(a) Show that if $-\pi/2 \leq x \leq \pi/2$

$$\sin nx = n \sin x \quad {}_2F_1\left(\frac{1+n}{2}, \frac{1-n}{2}; \frac{3}{2}; sin^2 x\right)$$

(b) Prove that

$$B(\lambda, \gamma - \lambda) \quad {}_2F_1(\alpha; \beta; \gamma; x) = \int_0^1 t^{\lambda-1} (1-t)^{\gamma-\lambda-1} \quad {}_2F_1(\alpha; \beta; \gamma; xt) dt$$

7.(a) Prove that zeros of the Legendre polynomials $P_n(x)$ are all real and lie between -1 and 1.

(b) Prove that

$$\int_{-1}^{1} x^2 P_{n+1}(x) P_{n-1}(x) dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$$

8.(a) Prove that $\frac{Q_n(x)}{P_n(x)} = \int_x^\infty \frac{dx}{(x^2-1)\{P_n(x)\}^2}$

(b) Prove that $nP_n(x) = xp_n'(x) - p_{n-1}'(x)$

where dashes indicates the derivatives with respect to x.

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M. Sc Mathematics Sem III

Paper-I

Functional Analysis -1

Assignment

Q.1 Prove that any convergent sequence in a metric space is Cauchy Sequence.

Q.2 Show that the function $d: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ defined by

$d(x, y) = \sqrt{|x - y|}$, $\forall\, x, y \in \mathbb{R}$ is a metric on $\mathbb{R}$.

Q.3 Prove that a subset of $\mathbb{R}^n$ is compact if and only if it is closed and bounded.

Q.4 State and prove Banach Contraction theorem.

Q.5 Show that $\mathbb{R}(\mathbb{R})$ is a Banach space under the norm $||x|| = |x|$.

Q.6 Prove that a linear operator $T: X \rightarrow Y$ between two normed linear spaces is bounded if and only if it is continuous.

Q.7 State and prove Closed Graph Theorem.

Q.8 State and prove Reisz's Lemma.

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M.Sc. (Mathematics) III Semester

Paper II Viscous Fluid Dynamics-I

Q. 1. Derive Equation of Continuity.

Q.2. Derive Navier Stokes Equation for viscous incompressible fluid motion.

Q.3. State and Prove Buckingham –π theorem.

Q.4. Write Short note on

1. Reynolds Number
2. Froude Number
3. Mach Number
4. Eckert Number
5. Prandtl Number.

Q. 5. Discuss the Flow in a circular tube.

Q. 6. Discuss the flow in a tube of rectangular cross-section.

Q.7. Define Stagnation Point and discuss Hiemenz Flow.

Q.8. Discuss the flow between two concentric rotating cylinders.

M.Sc. (Mathematics) III Semester

Paper IV Mathematical Programming -I

Assignment

Q. 1. Solve the following LPP by Revised Simplex Method

$$\begin{aligned} &Max.z = 2x_1 + x_2 \\ &subject\ to\ 3x_1 + 4x_2 \le 6 \\ &\qquad 6x_1 + x_2 \le 3 \\ &and\ x_1, x_2 \ge 0 \end{aligned}$$

Q.2. Using the bounded variable technique , solve the following LPP

$$\begin{aligned} &Max.z = x_2 + 3x_3 \\ &subject\ to\ x_1 + x_2 + x_3 \le 10 \\ &\qquad x_1 - 2x_3 \ge 0 \\ &\qquad 2x_1 - x_3 \le 10 \\ &and\ 0 \le x_1 \le 8; 0 \le x_2 \le 4\ ; x_3 \ge 0 \end{aligned}$$

Q.3. Use Gomory's Method to solve the following LPP:

$$\begin{aligned} &Max.z = 9x_1 + 10x_2 \\ &subject\ to\ x_1 + x_3 = 3 \\ &\qquad 2x_1 + 5x_2 + x_4 = 15 \\ &and\ x_1, x_2, x_3, x_4 \ge 0\ are\ integers \end{aligned}$$

Q.4. Use the Branch and Bound Algorithm to solve the following LPP:

$$\begin{aligned} &Max.z = 3x_1 + 4x_2 \\ &subject\ to\ 2x_1 + x_2 \le 6 \\ &\qquad 2x_1 + 3x_2 \le 9 \\ &and\ x_1, x_2 \ge 0\ and\ are\ integers \end{aligned}$$

Q. 5. Find an approximate optimal solution of the following Separable Programming Problem

$$
\begin{aligned}
&Max. z = x_1 + x_2{}^4 \\
&subject\ to\ 3x_1 + 2x_2^4 \leq 9 \\
&and\ \ x_1 \geq 0, x_2 \geq 0
\end{aligned}
$$

Q. 6. Solve the following Fractional Programming Problem:

$$
\begin{aligned}
&Max z = \frac{5x_1 + 3x_2}{5x_1 + 2x_2 + 1} \\
&s.t. \quad 3x_1 + 5x_2 \leq 15 \\
&\qquad 5x_1 + 2x_2 \leq 10 \\
&and \ \ x_1, x_2 \geq 0
\end{aligned}
$$

Q.7. Use Dynamic Programming to find maximum value of the product x_1, x_2, x_3, ----------------, x_n when $x_1+x_2+x_3+$----------------$+x_n=b$

x_1, x_2, x_3, ----------------, $x_n \geq 0$

Q.8. Solve the following LPP by Dynamic Programming:

$$
\begin{aligned}
&Max z = x_1 + 9x_2 \\
&subject to\ 2x_1 + x_2 \leq 25 \\
&\qquad\qquad x_2 \leq 11 \\
&and\ x_1 \geq 0, x_2 \geq 0
\end{aligned}
$$

M.Sc. Third Semester Assignment

MATHEMATICS

Third PAPER

Integral Transforms

1.(a) Find Fourier Cosine transform of $\frac{1}{1+x^2}$ and hence find Fourier sine transform of $\frac{x}{1+x^2}$

(b) Find the convolution of $f(x) = cosx$ and $g(x) = \exp(-a|x|), a > 0$

2.(a) Find Fourier transform of $f(x) = \frac{\sin ax}{x}, a > 0$

(b) Find $f(x)$ if its Fourier sin e transform is $\frac{e^{-ap}}{p}$. Hence deduce $f_s^{-1}\left\{\frac{1}{p}\right\}$.

3.(a) Prove that

$$M\left[\left(\frac{d}{dx}x\right)^m f(x);p\right] = (1-p)^m F(p)$$

where$F(p) = M[f(x);p]$ and m is positive integer.

(b) Prove that$M[(1+x)^{-a};p] = \frac{\Gamma(p)\Gamma(a-p)}{\Gamma(a)}, 0 < Re(p) < Re(a)$. Hence deduce that

$$M[(1+x)^{-1};p] = \prod cosec(\prod p), 0 < Re(p) < 1.$$

4.(a) Prove that $M\left[x^{\propto}\int_0^\infty u^\beta f(xu)g(u)du;p\right] = M[f(x);p+\propto]M[g(x);1+\beta-\propto-p]$

(b) Prove that $M^{-1}[\Gamma(p)F(1-p)x] = L[f(t);x]$, where $F(p) = M[f(x);p]$ andL$[f(t);x]$ is Laplace transform of $f(t)$.

5.(a) Find $L\{F(t)\}$, where $F(t) = \begin{Bmatrix} e^{t-a} & ,t>a \\ o & ,t<a \end{Bmatrix}$

(b) Find $L^{-1}\left\{\log\left(1+\frac{1}{p^2}\right)\right\}$

6.(a) Prove that $L\{J_0(x);p\} = \frac{1}{\sqrt{p^2+1}}$ and deduce $L\{J_0(ax);p\}$.

(b) Use the convolution theorem to find $L^{-1}\left\{\frac{p}{(p^2+a^2)^2}\right\}$

7.(a) Using the complex inversion formula for the Laplace transform, find inverse Laplace transform of $\frac{1}{(p^2+1)^2}$.

(b) Find the Hankel transform of $x^v e^{-ax}$, taking $xJ_v(px)$ as the kernal.

8.(a) Find Hankel transform of $\frac{cosax}{x}$ and $\frac{sinax}{x}$ taking $xJ_0(px)$ as the kernal.

(b) Find the Hankel transform of the function

(i) $\frac{d^2f(x)}{dx^2}+\frac{1}{x}\frac{df(x)}{dx}$, where $f(x)=\frac{e^{-ax}}{x}$ and $v=0$.

(ii) $\frac{df}{dx}, where\ f(x)=\frac{e^{-ax}}{x}$ and $v=1$.

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M.Sc. (Mathematics) III Semester

Paper V Relativistic Mechanics

Assignment

Q. 1. Obtain Lorentz Transformation Equations using Postulates of Special Relativity.

Q.2. Prove that the set of Special Lorentz Transformation satisfies group properties.

Q.3. Obtain the transformation formula for Lorentz Contraction Factor.

Q.4. If u and v are two velocities in the same direction and V is their resultant velocity given by

$$tan^{-1}\frac{V}{c} = tan^{-1}\frac{u}{c} + tan^{-1}\frac{v}{c}$$

Then deduce the law of composition of velocities from this equation.

Q. 5. Obtain transformation formula for momentum and energy. Hence show that $p^2 - E^2/c^2$ is Lorentz invariant and also prove that $E^2 = c^2(m_0^2c^2 + p^2)$ where the terms have their usual meaning.

Q. 6. Obtain the Expression for Relativistic Lagrangian.

Q.7. Explain space-like, time-like and light-like intervals with suitable example.

Q.8. Explain the principle of Equivalence with examples of Einstein Box experiment.

M.Sc. Third Semester Assignment
MATHEMATICS
SIXTH PAPER
Numerical Analysis

1. Find the multiple root of the equation

$$27x^5 + 27x^4 + 36x^3 + 28x^2 + 9x + 1 = 0$$

using Newton – Raphson method.

2. Find the root of the equations

$$\log_{10} x - x + 3 = 0$$

by Muller's method, taking initial approximations as

$$x_0 = 0.5 , \quad x_1 = 1 , \quad x_2 = 1.5$$

3. Use synthetic division and perform two iterations by Birge – Vieta method to find the smallest positive root of the equation $x^4 - 3x^3 + 3x^2 - 3x + \quad = 0$

4. Find all the roots of the polynomial $x^4 - x^3 + 3x^2 + x - 4 = 0$ Using the Graeffe's root squaring method.

5. Solve the state of equations:

$$x_1 - 13x_2 - 3x_3 = 49$$
$$5x_1 - 6x_2 + 17x_3 = 25$$
$$11x_1 + x_2 - 4x_3 = -31$$

using relaxation method (Perform five iterations)

6. Solve the system of equations.

$$4x_1 - x_2 = 1$$
$$-x_1 + 4x_2 - x_3 = 0$$
$$-x_2 + 4x_3 = 0$$

Using Cholesky's method.

7. Find all the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} 1 & \sqrt{2} & \\ \sqrt{2} & 3 & \sqrt{2} \\ & \sqrt{2} & 1 \end{bmatrix}$$

by Jacobi method.

8. Find all the eigen values of the matrix

$$A = \begin{bmatrix} 4 & 3 \\ 1 & \end{bmatrix}$$

using the Rutisausev method.

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